PROBLEM ON 2009 OCTOBER 21

MVHS NUMBER THEORY GROUP

If you tried to input the number

$$n = 2^{99999\ 99999}$$

into your calculator, then your calculator would give you an error message at best. This number is so absurdly huge (millions and millions of digits) we would never think about dividing this number! Or would we? Does 13 divide n evenly? If not, then what is the remainder of n when divided by 13?

As a hint, try writing 99999 99999 as a sum. For instance in base 2 we have

$$(99999\ 99999)_{10} = (10010\ 10100\ 00001\ 01111\ 10001\ 11111\ 1111)_2$$

Where the subscript indicates which base the number is written in. Or if you don't like base 2, we can write

$$99999\ 99999 = 10^{11} - 1$$

So we have two alternative ways of writing n. They are

$$n = 2^{99999\ 99999} = 2^{10^{11}-1}$$

$$n = 2^{99999\ 99999} = 2^{2^{33}+2^{30}+2^{28}+2^{26}+2^{19}+2^{17}+2^{16}+2^{15}+2^{14}+2^{13}+2^{9}+2^{8}+2^{7}+2^{6}+2^{5}+2^{4}+2^{3}+2^{2}+2^{1}+2^{0}}$$

How does this help you divide n? Remember that sums in the exponent translate into products of n. There is nothing special about 13 in this problem. Once you figure out how to divide n by 13 with remainder, you can use the same method to divide n by any number with remainder. This problem is worth **3 Points**. More importantly, if you can do this problem you will gain much insight into how the *encryption* side of the **RSA Algorithm** works.